RIEŠENIE - PRIEBEŽNÝ TEST - LS 2023/2024 - MATEMATIKA A

1.) $$\lim_{n\to\infty} \frac{n\cdot\left(\overbrace{\sqrt{n^3+2n}}^{a}-\overbrace{\sqrt{n^3-6n}}^{b}\right)}{\sqrt{4n+3}} \overset{F.3}{=} \lim_{n\to\infty} \frac{n\cdot\left(\sqrt{n^3+2n}-\sqrt{n^3-6n}\right)\cdot\left(\sqrt{n^3+2n}+\sqrt{n^3-6n}\right)}{\sqrt{4n+3}\cdot\left(\sqrt{n^3+2n}+\sqrt{n^3-6n}\right)}$$

$$\boxed{a^2-b=(a-b)(a+b)}$$

$$=\lim_{n\to\infty} \frac{n\cdot\left((n^3+2n)-(n^3-6n)\right)}{\sqrt{n(4+3/n)}\cdot\sqrt{n^3}\cdot\left(\sqrt{1+2/n^2}+\sqrt{1-6/n^2}\right)} = \lim_{n\to\infty}\frac{n\cdot(8n)}{\sqrt{n^4}\cdot\sqrt{(4+3/n)}\cdot\left(\sqrt{1+2/n^2}+\sqrt{1-6/n^2}\right)}$$

$$=\lim_{n\to\infty}\frac{n^2\cdot 8}{n^2\cdot\sqrt{4+\underset{\to 0}{3/n}}\cdot\left(\sqrt{1+\underset{\to 0}{2/n^2}}+\sqrt{1-\underset{\to 0}{6/n^2}}\right)} = \lim_{n\to\infty}\frac{8}{\underbrace{\sqrt{4}}_{=2}\cdot\underbrace{\left(\sqrt{1}+\sqrt{1}\right)}_{=2}} = \frac{8}{2\cdot 2} = \underline{\underline{2}}$$

(2 body)

2.) $$f(x)=e^{(3x^2+5x)}+\frac{x-5}{x^2}$$

$$x^2-2x^2+10x \qquad -x^2$$

$$\hookrightarrow f'(x)=e^{3x^2+5x}\cdot(6x+5)+\frac{1(x^2)-2x(x-5)}{x^4}=$$

$$=e^{3x^2+5x}\cdot(6x+5)+\frac{1}{x^2}-\frac{2x^2-10x}{x^4}=e^{3x^2+5x}(6x+5)+\frac{10x-x^2}{x^4}$$

$$=\underline{\underline{e^{3x^2+5x}(6x+5)+\frac{10-x}{x^3}}}$$

$$\hookrightarrow D_f=\mathbb{R}-\{0\}=(-\infty,0)\cup(0,\infty)$$

(2 body)

$$\hookrightarrow D_{f'}=\mathbb{R}-\{0\}=(-\infty,0)\cup(0,\infty)$$

3.) $$f(x)=\frac{4x-2}{x+1}\ ;\ f'(x)=\frac{4\cdot(x+1)-1\cdot(4x-2)}{(x+1)^2}=\frac{4x+4-4x+2}{(x+1)^2}=\underline{\underline{\frac{6}{(x+1)^2}}}$$

VZORČEK

$$\hookrightarrow \boxed{f'(x)=\left(\frac{h}{g}\right)'=\frac{h'\cdot g-g'\cdot h}{g^2}}$$

$$x_0=-2 \qquad \hookrightarrow \boxed{y=f'(x_0)\cdot(x-x_0)+f(x_0)}$$ ROVNICA „TEČNY" KU GRAFU FUNKCIE

$$\hookrightarrow f(x_0)=\frac{4(-2)-2}{-2+1}=\frac{-10}{-1}=\underline{\underline{+10}}\ ;\ f'(x_0)=\frac{6}{(-2+1)^2}=\underline{\underline{6}}$$

ROVNICA „TEČNY": $$y=6\cdot(x-(-2))+(-10)=6(x+2)-10=6x+12+10$$

$$\boxed{y=\underline{6x+22}}$$

BOD DOTYKU: $$(x_D,y_D)=\underline{\underline{(-2,10)}} \qquad \boxed{D=(-2,10)}$$

6 bodov

HYPERBOLA:

$$f(x) = \frac{4x-2}{x+1} = \frac{(4x-2+6)-6}{x+1} = \frac{4x+4-6}{x+1}$$

$$= \frac{4\cdot(x+1)}{\cancel{x+1}} - \frac{6}{x+1}$$

$$\boxed{f(x) = \frac{ax+b}{cx+d}}$$

STREDOVÝ TVAR $\rightarrow$

$$= \underbrace{4}_{y_S = 4} - \underbrace{\frac{6}{x+1}}_{x_S = -1}$$

$D_f = \mathbb{R} - \{-1\}$

$$\boxed{\begin{aligned} y_S &= \frac{a}{c} \\ x_S &= \frac{-d}{c} \end{aligned}}$$

$f(-4) = \frac{4(-4)-2}{-4+1} = \frac{-18}{-3} = 6$

$S = [-1, 4]$

$P_x: y = 0: \quad 0 = \frac{4x-2}{x+1} \Rightarrow 0 = 4x-2 \Rightarrow 4x = 2$

$x = \frac{2}{4} = \frac{1}{2}$

$P_x = [1/2, 0]$

$P_y: x = 0: \quad y = \frac{4\cdot 0 - 2}{0+1} = -\frac{2}{1} = -2 \Rightarrow P_y = [0, -2]$

PRE DOTYČNICU: $\boxed{y = 6x + 22} \Rightarrow P_x: y = 0: \quad 0 = 6x + 22$ $\quad P_{x_T} = [-3{,}66, 0]$

$6x = -22$

$x = -\frac{22}{6} = -3\,4/6$

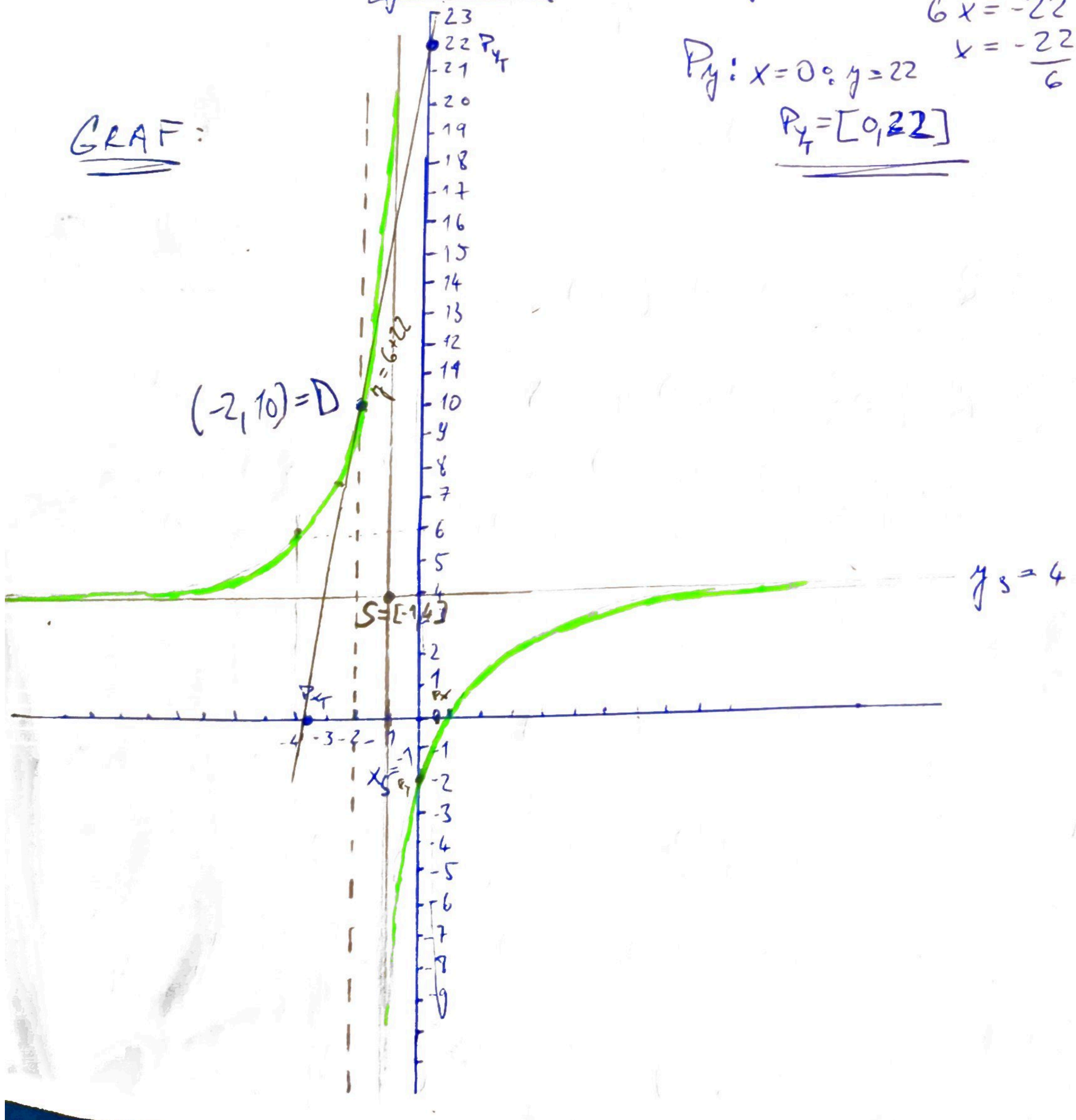
$= -3\,2/3$

$= -3{,}\overline{66}$

$P_y: x = 0: y = 22$

$P_{y_T} = [0, 22]$

GRAF:



4.) PRIEBEH FUNKCIE $f(x) = 4x^3 - x^4$

40 bodov

1.) $D_f = \mathbb{R} = (-\infty, \infty)$ Def. OBOR   lichá – sudá $\Rightarrow$ ani, ani

2.) lichá – sudá = ani sudá, ani lichá

LICHOSŤ/SUDOSŤ $f(-x) = 4\cdot(-x)^3 - (-x)^4 = -4(x)^3 - x^4 = -(4x^3 + x^4)$

$\Rightarrow f(-x) \neq f(x) \wedge f(x) \neq -f(x)$

3.) KDE JE fcia $f(x) > 0 \,/\, f(x) < 0$?

↳ $f(x) = 0$? : $4x^3 - x^4 = 0$

$x^3 \cdot (4 - x) = 0 \Rightarrow x_1 = 0 \wedge x_2 = 4$

	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$	VIZUALIZÁCIA
x^3	−	+	+	
$4-x$	+	+	−	
$f(x)$	−	+	−	

Funkcia $f(x)$ je kladná na intervale $(0,4)$ a $f(x) < 0$ na $(-\infty, 0) \cup (4, \infty)$.

4.) PRIESEČNÍKY

P_x: $y = 0$: $0 = 4x^3 - x^4 = x^3\cdot(4-x) \Rightarrow P_{x_1} = [0,0];\ [4,0] = P_{x_2}$

P_y: $x = 0$: $y = 4\cdot 0^3 - 0^4 = 0 \Rightarrow P_y = [0,0]$

5.) LIMITY V „KB": KRAJNÉ BODY (KB) $= \{-\infty, \infty\}$

$\lim_{x \to \infty} 4x^3 - x^4 = \lim_{x \to \infty} x^3 \cdot (4 - x) = -\infty$

$\lim_{x \to -\infty} 4x^3 - x^4 = \lim_{x \to -\infty} x^3 \cdot (4 - x) = -\infty$

6.) DERIVÁCIA $f(x)$ a NB (nulové body derivácie) → STACIONÁRNE BODY

$f'(x) = 4\cdot 3x^2 - 4x^3 = 12x^2 - 4x^3$

$f'(x) = 0$: $12x^2 - 4x^3 = 0 \Rightarrow x^2 \cdot (12 - 4x) = 0 \Rightarrow x_1 = 0 \wedge 12 - 4x = 0$

$12 = 4x$

$x_2 = 3$

7.) INTERVALY MONOTÓNNOSTI

	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$	VIZUALIZÁCIA
x^2	+	+	+	
$12-4x$	+	+	−	
$f'(x)$	+	+	−	

$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
$f'(x) > 0$	$f'(x) > 0$	$f'(x) < 0$
$f(x)$ rast. ↗	$f(x)$ rast. ↗	$f(x)$ kles. ↘

Funkcia $f(x)$ je rast. na $(-\infty, 0) \cup (0, 3)$ a kles. na $(3, \infty)$.

8. LOKÁLNE A GLOBÁLNE EXTRÉMY

+ DRUHÁ DERIVÁCIA

$$f''(x) = (12x^2-4x^3)' = 12\cdot 2x - 4\cdot 3x^2 = 12x\cdot(2-x) = 24x - 12x^2$$

$$f''(x)=0: \quad 12x\cdot(2-x)=0 \Rightarrow x_1=0 \wedge x_2=2$$ (NB 2. derivácie)

↳ v bode $(0,0)$ nemôže byť extrém, pretože 2. der. sa tu rovná opäť nule. V bode $x=3$, však: $f''(3) = \underbrace{12\cdot(3)}_{36}\cdot\underbrace{(2-3)}_{(-1)} = \underline{\underline{-36<0}}$

↳ v STAC. BODE $x=3$ je $f''(3)<0 \Rightarrow$ MAXIMUM. (videli sme už z tabuľky).

↳ vďaka $\lim_{x\to\pm\infty} f(x) = -\infty$ vieme, že je to GLOBÁLNE MAXIMUM.

9. OBOR HODNÔT H_f:

z $\lim_{x\to\pm\infty} f(x) = -\infty$ a z $f(x=3) = 4\cdot(3)^3-(3)^4 = 4\cdot 27 - 81 = 108-81 = \underline{\underline{27}}$

↳ $H_f = (-\infty, 27)$

10. ASYMPTOTY v $\pm\infty$:

↳ zvislé: nemáme, lebo nemáme bod x_0, v kt. by $\lim_{x\to x_0^-} f(x) = \pm\infty$ $\wedge \lim_{x\to x_0^+} f(x) = \pm\infty$.

↳ šikmé: $\lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{4x^3-x^4}{x} = \lim_{x\to\pm\infty} 4x^2-x^3 = \lim_{x\to\pm\infty} x^2\cdot(4-x) = \pm\infty \notin \mathbb{R}$

↳ nemá šikmú asymptotu

11. KONVEXITA/KONKÁVITA A INFLEXNÉ BODY:

INFLEXNÝ BOD: $\exists f''(x) = 0 \wedge f'''(x) \neq 0$

$$f''(x) = 12x\cdot(2-x) \Rightarrow \text{NB: } x_1=0 \wedge x_2=2$$

$(-\infty, 0)$	$(0,2)$	$(2,\infty)$
$f''(x)<0$	$f''(x)>0$	$f''(x)<0$
KONKÁVNA	KONVEXNÁ	KONKÁVNA

→ KANDIDÁTI NA INFLEXNÝ BOD:

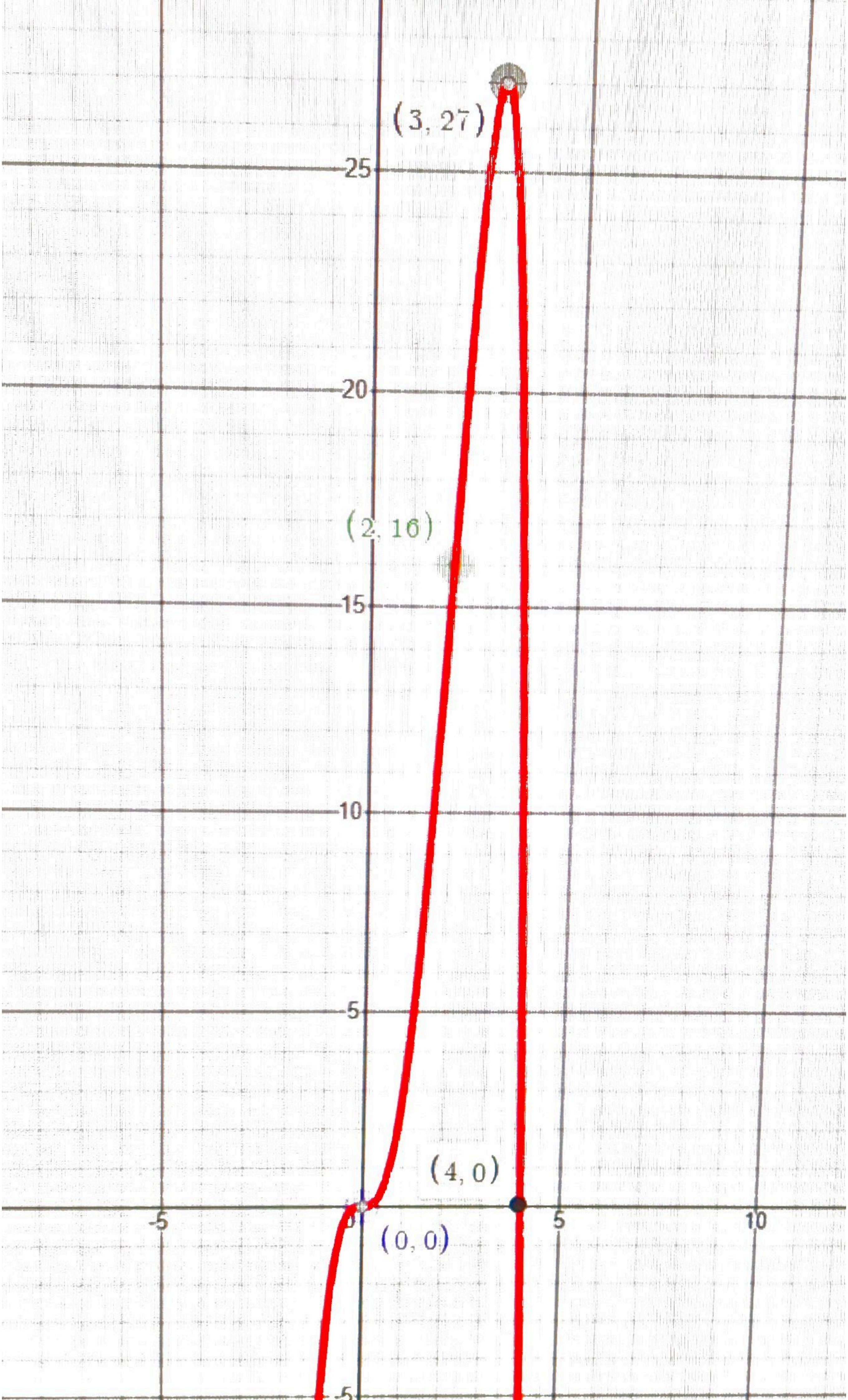
$x_1=0$, $x_2=2$: $f'''(x) = 24-24x$

$f'''(0) = 24-24\cdot 0 = 24 \neq 0$

$f'''(2) = 24-24\cdot 2 = -24 \neq 0$

Funkcia $f(x)$ má 2 inflexné body: v $[0,0]$ a v $[2,16]$

$(f(2) = 4\cdot(2)^3-(2)^4 = 4\cdot 8 - 16 = 32-16 = 16)$


(3, 27)
25
20
(2, 16)
15
10
5
(4, 0)
-5
0
5
10
(0, 0)
-5